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Consequences of symmetries and consistency relations in the large-scale structure of the universe for non-local bias and modified gravity

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Abstract

Consistency relations involving the soft limit of the $(n+1)$-correlation functions of dark matter and galaxy overdensities can be obtained, both in real and redshift space, thanks to the symmetries enjoyed by the Newtonian equations of motion describing the dark matter and galaxy fluids coupled through gravity. We study the implications of such symmetries for the theory of galaxy bias and for the theories of modified gravity. We find that the invariance of the fluid equations under a coordinate transformation that induces a long-wavelength velocity constrains the bias to depend only on a set of invariants, while the symmetry of such equations under Lifshitz scalings in the case of matter domination allows one to compute the time-dependence of the coefficients in the bias expansion. We also find that theories of modified gravity which violate the equivalence principle induce a violation of the consistency relation which may be a signature for their observation. Thus, given adiabatic Gaussian initial conditions, the observation of a deviation from the consistency relation for galaxies would signal a breakdown of the so-called non-local Eulerian bias model or the violation of the equivalence principle in the underlying theory of gravity.

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1. Introduction

Symmetries play a crucial role in understanding the properties of a physical system and they have turned out to be quite useful in characterizing the cosmological perturbations generated during a de Sitter stage [1]. Since the de Sitter isometry group SO(1, 4) acts like conformal group on $\mathbb{R}^3$ when the fluctuations are on super-Hubble scales, the correlators of scalar fields, which are not the inflaton, are constrained by conformal invariance [2–6]. The fact that the de Sitter isometry group acts as conformal group on the three-dimensional Euclidean space on super-Hubble scales can be also used to predict the shape of the correlators involving the inflaton and vector fields [7]. Furthermore, if the inflationary perturbations are generated in single-field models of inflation, there exist conformal consistency relations among the inflationary correlators [8–15].

Consistency relations involving the soft limit of the $(n + 1)$-correlation functions of matter and galaxy overdensities have also been proposed by investigating the symmetries enjoyed by the Newtonian equations of motion of the non-relativistic dark matter and galaxy fluids coupled to gravity [16,17]. These consistency relations have been recently generalized to the relativistic limit [18] (see also [19]), based on the observation that a long mode, in single-field models of inflation, reduces to a diffeomorphism since its freezing during inflation all the way until the late universe, even when the long mode is inside the horizon (but out of the sound horizon).

The large-scale consistency relations have the virtue of being true also for the galaxy overdensities, independently of the bias between galaxy and dark matter. As such, they may serve as a guidance in building up a bias theory. Indeed, we will argue that the non-local Eulerian bias model can be seen as being built of quantities which are invariant under the symmetries enjoyed by the Newtonian fluid equations. Furthermore, they might be useful in testing theories of modified gravity where extra degrees of freedom appear mediating extra long-range forces (other than the gravitational one) and possibly leading to a violation of the Equivalence Principle (EP) in the late universe and therefore to a violation of the consistency relation. In fact, assuming adiabatic Gaussian initial conditions, an observed violation of the consistency relations would either indicate a breakdown of the non-local Eulerian bias model (and also the presence of terms in the effective fluid equations for galaxies that break the aforementioned symmetries), or a violation of the EP in the underlying theory of gravity.

It is in the spirit of exploring these topics that in this paper we aim to investigate what the large-scale consistency relations may tell us about the galaxy bias and how they can be used to scrutinize modified gravity theories. In particular, we will show that the symmetries leading to the consistency relations allow the presence of what is commonly dubbed non-local bias, that is a relation between the galaxy and the dark matter overdensities which is not a simple function of the local dark matter abundance. We will identify a series of invariants (with respect to the symmetries) which should appear in the galaxy bias expansion, precisely because they are allowed by the symmetries of the problem. Furthermore, we will investigate under which conditions the consistency relations are valid in the case in which a modification of gravity is attained far in the infrared on cosmological scales.

The paper is organized as follows. In Section 2 we discuss the symmetries of the non-relativistic fluid equations for both dark matter and galaxies and we derive galaxy consistency relations for the n-point correlators of short wavelength modes in the background of a long wavelength mode perturbation. In Section 3 we provide the invariants under the symmetries of the galaxy and dark matter fluids and we discuss their implications for the non-local bias. We also check that the galaxy consistency relation holds at tree- and one-loop level in the bias model. In Section 4 we show how to extend the galaxy consistency relations to redshift space where actual experiments are made. In Section 5 we discuss the consequences of the symmetries for the theories of modified gravity and how such modifications are imprinted in the $(n+1)$-point correlators in the squeezed limit. Finally, Section 6 presents our conclusions.

2. Symmetries and consistency relation of galaxy correlation functions in real space

Galaxies (or more precisely, some population thereof), once formed, obey the following equations on sub-Hubble scales

$$\frac{\partial \delta_{\rm g}(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot\big[\big(1+\delta_{\rm g}(\vec{x},\tau)\big)\vec{v}_{\rm g}(\vec{x},\tau)\big] = 0, \tag{2.1}$$

$$\frac{\partial \vec{v}_{\rm g}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}_{\rm g}(\vec{x},\tau) + \big[\vec{v}_{\rm g}(\vec{x},\tau)\cdot\vec{\nabla}\big]\vec{v}_{\rm g}(\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau), \tag{2.2}$$

$$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau), \tag{2.3}$$

where we have denoted by $\vec{x}$ the comoving spatial coordinates, $\tau = \int {\rm d}t/a$ the conformal time, a the scale factor in the FRW metric and $\mathcal{H} = {\rm d}\ln a/{\rm d}\tau$ is the conformal expansion rate. In addition, $\delta(\vec{x},\tau) = (\rho(\vec{x},\tau)/\bar{\rho} - 1)$ is the overdensity over the mean matter density $\bar{\rho}$, $\delta_{\rm g}(\vec{x},\tau)$ and $\vec{v}_{\rm g}(\vec{x},\tau)$ are the galaxy overdensity and peculiar velocity, and $\Phi(\vec{x},\tau)$ is the gravitational potential due to density fluctuations. Finally $\Omega_{\rm m} = 8\pi G\bar{\rho}a^2/3\mathcal{H}^2$ is the density parameter. Eq. (2.1) assumes number conservation [20]. Eventually, one would like to go beyond the treatment presented here in order to account for phenomena like formation and merging, which could be done for example by adding a source term to the right hand side of Eq. (2.1).

Dark matter is described by a similar set of non-relativistic fluid equations in the presence of gravity

$$\frac{\partial \delta(\vec{x},\tau)}{\partial \tau} + \vec{\nabla}\cdot\big[\big(1+\delta(\vec{x},\tau)\big)\vec{v}(\vec{x},\tau)\big] = 0, \tag{2.4}$$

$$\frac{\partial \vec{v}(\vec{x},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}(\vec{x},\tau) + \big[\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}\big]\vec{v}(\vec{x},\tau) = -\vec{\nabla}\Phi(\vec{x},\tau), \tag{2.5}$$

$$\nabla^2\Phi(\vec{x},\tau) = \frac{3}{2}\Omega_{\rm m}\mathcal{H}^2(\tau)\delta(\vec{x},\tau). \tag{2.6}$$

Following Ref. [16], one can show that in ΛCDM cosmology the set of Eqs. (2.1)–(2.3) and (2.4)–(2.6) is invariant under the transformations (for a generic vector $\vec{n}(T)$)

$$\tau' = \tau, \qquad \vec{x}' = \vec{x} + \vec{n}(T), \tag{2.7}$$

where

$$T(\tau) = \frac{1}{a(\tau)}\int^{\tau} {\rm d}\eta\, a(\eta), \tag{2.8}$$

provided that one transforms the fields as follows

$$\delta'_{\rm g}(\vec{x},\tau) = \delta_{\rm g}\big(\vec{x}',\tau'\big), \tag{2.9}$$

$$\vec{v}'_{\rm g}(\vec{x},\tau) = \vec{v}_{\rm g}\big(\vec{x}',\tau'\big) - \dot{\vec{n}}(T), \tag{2.10}$$

$$\delta'(\vec{x},\tau) = \delta\big(\vec{x}',\tau'\big), \tag{2.11}$$

$$\vec{v}'(\vec{x},\tau) = \vec{v}\big(\vec{x}',\tau'\big) - \dot{\vec{n}}(T), \tag{2.12}$$

$$\Phi'(\vec{x},\tau) = \Phi\big(\vec{x}',\tau'\big) - \big(\mathcal{H}\dot{\vec{n}}(T) + \ddot{\vec{n}}(T)\big)\cdot\vec{x}. \tag{2.13}$$

This is true even if we do not set $\vec{v}_{\rm g}(\vec{x},\tau)=\vec{v}(\vec{x},\tau)$, that is if we do not assume that the galaxy peculiar velocity is unbiased. Note that if one adds a source term to the right hand side of Eq. (2.1) to account for the change of the number density of galaxies in time, and such a source term depends only on quantities which transform as scalars, the equations of motion are still invariant under these transformations.

Consider the n-point correlation function of short modes of the density contrast. The symmetries of the Newtonian fluid equations imply, for instance, that

$$\left\langle\delta'_{\rm g}(\vec{x}_1)\cdots\delta'_{\rm g}(\vec{x}_n)\right\rangle=\left\langle\delta_{\rm g}(\vec{x}_1)\cdots\delta_{\rm g}(\vec{x}_n)\right\rangle=\left\langle\delta_{\rm g}\left(\vec{x}'_1\right)\cdots\delta_{\rm g}\left(\vec{x}'_n\right)\right\rangle. \tag{2.14}$$

The points are supposed to be contained in a sphere of radius R much smaller than the long wavelength mode of size $\sim 1/q$ and centered at the origin of the coordinates. The non-relativistic equations of motion are invariant under the generic transformation $\tau\to\tau$ and $\vec{x}\to\vec{x}+\vec{n}(T(\tau))$. This means that we can generate a long wavelength mode for the dark matter velocity perturbation $\vec{v}_L(\tau,\vec{0})$ just by choosing properly the vector $\vec{n}(\tau)$

$$\vec{n}(\tau)=-\int^{\tau}{\rm d}\eta\,\vec{v}_L(\eta,\vec{0})+\mathcal{O}\left(qRv_L^2\right). \tag{2.15}$$

In other words, the correlator of the short wavelength modes in the background of the long wavelength mode perturbation should satisfy the relation [16]

$$\left\langle\delta_{\rm g}(\tau_1,\vec{x}_1)\delta_{\rm g}(\tau_2,\vec{x}_2)\cdots\delta_{\rm g}(\tau_n,\vec{x}_n)\right\rangle_{v_L}=\left\langle\delta_{\rm g}\left(\tau'_1,\vec{x}'_1\right)\delta_{\rm g}\left(\tau'_2,\vec{x}'_2\right)\cdots\delta_{\rm g}\left(\tau'_n,\vec{x}'_n\right)\right\rangle. \tag{2.16}$$

This is nothing else that the statement that the effect of a physical long wavelength galaxy velocity perturbation onto the short modes should be indistinguishable from the long wavelength mode velocity generated by the transformation with $\delta x^i=n^i(\tau)$. In momentum space one therefore obtains

$$\left\langle\delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle_{q\to 0}=\left\langle\delta_{\rm g}(\vec{q},\tau)\left\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle_{v_L}\right\rangle. \tag{2.17}$$

The variation of the n-point correlator under the infinitesimal transformation is given by

$$\begin{aligned}\delta_n\left\langle\delta_{\rm g}(\tau_1,\vec{x}_1)\cdots\delta_{\rm g}(\tau_n,\vec{x}_n)\right\rangle&=\int\frac{{\rm d}^3\vec{k}_1}{(2\pi)^3}\cdots\frac{{\rm d}^3\vec{k}_n}{(2\pi)^3}\left\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle\\&\quad\times\sum_{a=1}^{n}\delta x_a^i\left(ik_a^i\right)e^{i(\vec{k}_1\cdot\vec{x}_1+\cdots\vec{k}_n\cdot\vec{x}_n)}\\&=\int\frac{{\rm d}^3\vec{k}_1}{(2\pi)^3}\cdots\frac{{\rm d}^3\vec{k}_n}{(2\pi)^3}\left\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle\\&\quad\times\sum_{a=1}^{n}n^i(\tau_a)\left(ik_a^i\right)e^{i(\vec{k}_1\cdot\vec{x}_1+\cdots\vec{k}_n\cdot\vec{x}_n)}.\end{aligned} \tag{2.18}$$

Then we find that

$$\begin{aligned}\left\langle\delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle_{q\to 0}&=\left\langle\delta_{\rm g}(\vec{q},\tau)\left\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle_{v_L}\right\rangle\\&=i\sum_{a=1}^{n}\left\langle\delta_{\rm g}(\vec{q},\tau)n^i(\tau_a)\right\rangle k_a^i\left\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta_{\rm g}(\vec{k}_n,\tau_n)\right\rangle.\end{aligned} \tag{2.19}$$

In a ΛCDM model we have

$$\int^{\tau} \mathrm{d}\eta\, \vec{v}_L(\vec{q},\eta) = i\frac{q^i}{q^2}\int^{\tau} \mathrm{d}\eta\, \mathcal{H}\frac{1}{\mathcal{H}}\frac{\mathrm{d}\ln D(\eta)}{\mathrm{d}\eta}\frac{D(\eta)}{D(\eta_{\mathrm{in}})}\delta_L(\vec{q},\eta_{\mathrm{in}}) = i\frac{\vec{q}}{q^2}\delta_L(\vec{q},\tau), \tag{2.20}$$

where $D(\tau)$ is the linear growth factor and δ is the dark matter overdensity. We thus obtain the consistency relation

$$\boxed{\begin{aligned} &\big\langle \delta_{\mathrm{g}}(\vec{q},\tau)\delta_{\mathrm{g}}(\vec{k}_1,\tau_1)\cdots\delta_{\mathrm{g}}(\vec{k}_n,\tau_n)\big\rangle'_{q\to 0} \\ &\quad = -\big\langle \delta^L_{\mathrm{g}}(\vec{q},\tau)\delta_L(\vec{q},\tau)\big\rangle' \sum_{a=1}^{n}\frac{D(\tau_a)}{D(\tau)}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\big\langle \delta_{\mathrm{g}}(\vec{k}_1,\tau_1)\cdots\delta_{\mathrm{g}}(\vec{k}_n,\tau_n)\big\rangle' \end{aligned}} \tag{2.21}$$

where the primes indicate that one should remove the Dirac delta's coming from the momentum conservation. Notice that, if the correlators are computed all at equal times, the right-hand side of Eq. (2.21) vanishes by momentum conservation and the $1/q^2$ infrared divergence will not appear when calculating invariant quantities. For the three-point correlator, we obtain

$$\begin{aligned} \big\langle \delta_{\mathrm{g}}(\vec{q},\tau)\delta_{\mathrm{g}}(\vec{k}_1,\tau_1)\delta_{\mathrm{g}}(\vec{k}_2,\tau_2)\big\rangle'_{q\to 0} &= -\big\langle \delta^L_{\mathrm{g}}(\vec{q},\tau)\delta_L(\vec{q},\tau)\big\rangle'\left(\frac{D(\tau_1)}{D(\tau)}-\frac{D(\tau_2)}{D(\tau)}\right) \\ &\quad\times\frac{\vec{q}\cdot\vec{k}_1}{q^2}\big\langle \delta_{\mathrm{g}}(\vec{k}_1,\tau_1)\delta_{\mathrm{g}}(\vec{k}_2,\tau_2)\big\rangle'. \end{aligned} \tag{2.22}$$

Similarly, the dark matter correlators of the short wavelength modes in the background of the long wavelength mode perturbation should satisfy the relation

$$\big\langle \delta(\tau_1,\vec{x}_1)\delta(\tau_2,\vec{x}_2)\cdots\delta(\tau_n,\vec{x}_n)\big\rangle_{v_L} = \big\langle \delta\big(\tau'_1,\vec{x}'_1\big)\delta\big(\tau'_2,\vec{x}'_2\big)\cdots\delta\big(\tau'_n,\vec{x}'_n\big)\big\rangle, \tag{2.23}$$

leading to [16–18]

$$\begin{aligned} &\big\langle \delta(\vec{q},\tau)\delta(\vec{k}_1,\tau_1)\cdots\delta(\vec{k}_n,\tau_n)\big\rangle'_{q\to 0} \\ &\quad = -P_{\delta_{\mathrm{lin}}}(q,\tau)\sum_{a=1}^{n}\frac{D(\tau_a)}{D(\tau)}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\big\langle \delta(\vec{k}_1,\tau_1)\cdots\delta(\vec{k}_n,t_n)\big\rangle', \end{aligned} \tag{2.24}$$

where $P_{\delta_{\mathrm{lin}}}(q,\tau) = (D(\tau)/D(\tau_{\mathrm{in}}))^2 P_{\delta_{\mathrm{lin}}}(q,\tau_{\mathrm{in}})$ is the linear matter power spectrum. For the three-point correlator, we obtain

$$\begin{aligned} &\big\langle \delta(\vec{q},\tau)\delta(\vec{k}_1,\tau_1)\delta(\vec{k}_2,\tau_2)\big\rangle'_{q\to 0} \\ &\quad = -P_{\delta_{\mathrm{lin}}}(q,\tau)\left(\frac{D(\tau_1)}{D(\tau)}-\frac{D(\tau_2)}{D(\tau)}\right)\frac{\vec{q}\cdot\vec{k}_1}{q^2}\big\langle \delta(\vec{k}_1,\tau_1)\delta(\vec{k}_2,\tau_2)\big\rangle'. \end{aligned} \tag{2.25}$$

Once more, we stress that these relations are valid beyond linear order for the short wavelength modes which might well be in the non-perturbative regime.

3. Consequences of the symmetries for the galaxy bias theory: non-local bias

As the galaxy and dark matter overdensities equations of motion (2.1)–(2.3) and (2.4)–(2.6) are invariant under the set of transformations (2.7)–(2.13), an immediate consequence is that one can construct scalar quantities, i.e. quantities $S(\vec{x},\tau)$ which upon the transformation (2.7) are such that

$$S'(\vec{x},\tau) - S(\vec{x},\tau) = \vec{n}\cdot\vec{\nabla}S(\vec{x},\tau). \tag{3.1}$$

As the spatial gradients remain invariant, $\vec{\nabla} = \vec{\nabla}'$, one can easily realize that there are the following scalar quantities in the dark matter sector at our disposal

$$\boxed{\begin{aligned}&\delta(\vec{x},\tau), \quad s_{ij}(\vec{x},\tau) = \partial_i\partial_j\Phi(\vec{x},\tau) - \frac{\delta_{ij}}{2}\Omega_{\rm m}\mathcal{H}^2\delta(\vec{x},\tau),\\ &t_{ij}(\vec{x},\tau) = \partial_i v_j(\vec{x},\tau) - \frac{\delta_{ij}}{3}\theta(\vec{x},\tau) - \frac{2f}{3\Omega_{\rm m}\mathcal{H}}s_{ij}(\vec{x},\tau)\end{aligned}} \tag{3.2}$$

where $\theta(\vec{x},\tau) = \vec{\nabla}\cdot\vec{v}(\vec{x},\tau)$, $f = \mathrm{d}\ln D/\mathrm{d}\ln a$ (with $D(a)$ is the growth factor as a function of the scale factor a), we have removed the trace part from $\partial_i\partial_j\Phi(\vec{x},\tau)$, which is nothing else than the dark matter overdensity $\delta(\vec{x},\tau)$, and $t_{ij}(\vec{x},\tau)$ is vanishing at first-order in perturbation theory. Notice that these quantities are scalars beyond the linear perturbation theory as the symmetries identified in the previous section are valid at any order in perturbation theory. These symmetries are larger than the Galilean group identified in Ref. [21] for the large-scale dynamics. Furthermore, upon constructing the invariant operators

$$D^{v}_{\tau} = \frac{\partial}{\partial\tau} + \vec{v}(\vec{x},\tau)\cdot\vec{\nabla} \quad \text{and} \quad D^{v_{\rm g}}_{\tau} = \frac{\partial}{\partial\tau} + \vec{v}_{\rm g}(\vec{x},\tau)\cdot\vec{\nabla}, \tag{3.3}$$

one can construct two more scalar quantities

$$\vec{\nabla}\Phi(\vec{x},\tau) + D^{v}_{\tau}\vec{v}(\vec{x},\tau) + \mathcal{H}\vec{v}(\vec{x},\tau) \quad \text{and} \quad \vec{\nabla}\Phi(\vec{x},\tau) + D^{v_{\rm g}}_{\tau}\vec{v}_{\rm g}(\vec{x},\tau) + \mathcal{H}\vec{v}_{\rm g}(\vec{x},\tau), \tag{3.4}$$

but they are nothing else than the momentum conservation quantities for the dark matter and the galaxy, respectively. They identically vanish on-shell and therefore are trivial.

The set of invariants (3.2) are useful in constructing a galaxy bias theory which goes beyond the local bias model [22]. In the latter the galaxy overdensity $\delta_{\rm g}(\vec{x},\tau)$ is written as a completely general function $f[\delta(\vec{x},\tau)]$ of the mass density perturbation $\delta(\vec{x},\tau)$, and then the function is Taylor expanded, with the unknown coefficients in the series becoming the bias parameters

$$\delta_{\rm g}(\vec{x},\tau) = f\big[\delta(\vec{x},\tau)\big] = b_1\delta(\vec{x},\tau) + \frac{b_2}{2}\delta^2(\vec{x},\tau) + \cdots. \tag{3.5}$$

This local expansion, even though it is consistent with the first invariant of the list (3.2), is expected to be valid only on very large scales and small times: as the symmetry dynamics allows the presence of more scalar quantities, there is no reason why they should not be generated along the subsequent evolution. This logic is the same which applies in quantum field theory for operators: even though some of them are not present in the tree-level Lagrangian, they will appear at a certain order in perturbation theory unless they are forbidden by symmetry arguments. Therefore, assuming homogeneity and isotropy, one would expect a more general bias model of the form (where the coefficients should be intended to be the renormalized ones [23])

$$\begin{aligned}\delta_{\rm g}(\vec{x},\tau) = {}& b_1(\tau)\delta(\vec{x},\tau) + \frac{b_2(\tau)}{2}\delta^2(\vec{x},\tau) + c_{\nabla^2}(\tau)\nabla^2\delta(\vec{x},\tau) + c_{s^2}(\tau)s_{ij}(\vec{x},\tau)s^{ij}(\vec{x},\tau)\\ &+ c_{s^2\nabla}\partial_k s_{ij}(\vec{x},\tau)\partial^k s^{ij}(\vec{x},\tau) + c_{s^2\nabla^2}\nabla^2\big(s_{ij}(\vec{x},\tau)s^{ij}(\vec{x},\tau)\big)\\ &+ c_{s^2\nabla^4}(\tau)\nabla^2 s_{ij}(\vec{x},\tau)\nabla^2 s^{ij}(\vec{x},\tau)\\ &+\cdots,\end{aligned} \tag{3.6}$$

and the dots stand for the various other terms one can construct out of $s_{ij}(\vec{x},\tau)$ and gradients. We see that an unavoidable consequence of the symmetries of the problem is that the bias model is a non-local bias model [23–26]; in fact the non-local expansion (3.6) has been first proposed in Ref. [23] where the same invariants have been employed based on general arguments on the homogeneous gravitational field and dark matter velocity. Some comments are in order:

- The series does not contain a piece proportional to the gravitational potential $\Phi(\vec{x},\tau)$: it is simply forbidden by the symmetries of the problem as $\Phi(\vec{x},\tau)$ alone is not a scalar quantity.
- The non-local bias expansion (3.6) is not dictated solely by rotational invariance. Instead it is the more generic symmetry (2.7) together with isotropy which fixes the form of the expansion.
- The fluid equations during the matter-dominated period are also invariant under Lifshitz scalings of the form [16,27,28]

$$\tau' = \lambda^z \tau, \qquad \vec{x}' = \lambda \vec{x}, \tag{3.7}$$

$$\delta'(\vec{x},\tau) = \delta\big(\vec{x}',\tau'\big), \tag{3.8}$$

$$\delta'_{\rm g}(\vec{x},\tau) = \delta_{\rm g}\big(\vec{x}',\tau'\big), \tag{3.9}$$

$$\vec{v}'_{\rm g}(\vec{x},\tau) = \lambda^{z-1}\vec{v}\big(\vec{x}',\tau'\big), \tag{3.10}$$

$$\vec{v}'(\vec{x},\tau) = \lambda^{z-1}\vec{v}_{\rm g}\big(\vec{x}',\tau'\big), \tag{3.11}$$

$$\Phi'(\vec{x},\tau) = \lambda^{2(z-1)}\Phi\big(\vec{x}',\tau'\big), \tag{3.12}$$

  for a generic Lifshitz weight z and

$$\frac{\partial}{\partial\tau} = \lambda^z \frac{\partial}{\partial\tau'}, \qquad \vec{\nabla} = \lambda\vec{\nabla}'. \tag{3.13}$$

  Therefore, the Lifshitz weights of the bias coefficients should be

$$\begin{aligned} &[b_1] = [b_2] = 0, \qquad [c_{\nabla^2}] = 2, \qquad [c_{s^2}] = -4z, \\ &[c_{s^2\nabla^2}] = [c_{s^2\nabla}] = -2-4z, \qquad [c_{s^2\nabla^4}] = -4-4z. \end{aligned} \tag{3.14}$$

  These Lifshitz weights fix the time-behaviour of the corresponding coefficients for the growing mode. The fact that the Lifshitz weights of b_1 and b_2 are vanishing tell us that their growing mode is constant in time. Indeed, it is well-known that at large times the system experiences the so-called debiasing: b_1 converges to unity and b_2 goes to zero. Furthermore, the Lifshitz weights fix the corresponding time-behaviour of the remaining bias coefficients in their growing modes: c_{∇^2}, c_{s^2}, $c_{s^2\nabla^2}$ and $c_{s^2\nabla^4}$ should scale as $\tau^{2/z}$, τ^4, $\tau^{(4z+2)/z}$ and $\tau^{(4z+4)/z}$, respectively. In particular, if one matches with the linear power spectrum of dark matter with spectral index n, one finds $z = 4/(3+n) \simeq 1$ [21]. This explains why the non-local bias coefficients increase with time during the matter-dominated period. Furthermore, if one expresses the non-local invariant $s_{ij}(\vec{x},\tau)s^{ij}(\vec{x},\tau)$ at second-order in terms of the product of the linear overdensities, one finds that the Lifshitz symmetry imposes that the overall time scaling is τ^{-2} in a matter-dominated universe (once one goes to momentum space). This is precisely the scaling found in Ref. [29] and leads to the so-called debiasing, that is at late times the bias converges to unity and matter and galaxy density fields agree.
- As we already mentioned, galaxies form at a range of redshifts and merge. So it would be interesting to extend our results to the more realistic case when the number density of

galaxies changes with redshift due to some arbitrary source including the effects of galaxy formation and merging. However, if the effective source is a function of the scalar functions described above then our symmetry considerations will apply to this more complete galaxy description too. For instance, in Ref. [26] it was assumed that the effective source was of the form $A(\tau)j(\rho)$, where $A(\tau)$ parametrizes the epoch of galaxy formation and $j(\rho)$ the effects of dark matter on galaxy formation and merging. In such a case the symmetry (2.7)–(2.13) holds.

- If the fluid equations are not invariant under the set of transformations (2.7)–(2.13), as it happens for example in some modified theories of gravity to be discussed below, one expects other terms to appear in the bias expansion as the bias is scale-dependent. The possibility of testing the Poisson equation with a scale-dependent bias was discussed in [31].

3.1. Consequences of the symmetries for the galaxy bias theory: independence from the smoothing scale

The galaxy consistency relation also holds for smoothed quantities as the smoothing operation commutes with the coordinate transformation (2.7). Indeed, suppose we perform a smoothing operation with a window function around a sphere of radius R_L

$$\delta_{R_L}(\vec{x}) = \int \mathrm{d}^3 y\, W\big(|\vec{y}-\vec{x}|, R_L\big)\delta(\vec{y}), \tag{3.15}$$

where W is the appropriate window function. Then we have

$$\begin{aligned}\delta_{R_L}\big(\vec{x}'\big) &= \int \mathrm{d}^3 y\, W\big(\big|\vec{y}-\vec{x}'\big|, R_L\big)\delta(\vec{y}) = \int \mathrm{d}^3 y'\, W\big(\big|\vec{y}'-\vec{x}'\big|, R_L\big)\delta\big(\vec{y}'\big)\\ &= \int \mathrm{d}^3 y\, W\big(|\vec{y}-\vec{x}|, R_L\big)\delta'(\vec{y})\\ &= \delta'_{R_L}(\vec{x}),\end{aligned} \tag{3.16}$$

where in the last passage we have made use of the properties $\mathrm{d}^3 y' = \mathrm{d}^3 y$ and $(\vec{y}' - \vec{x}') = (\vec{y} - \vec{x})$. This has an important consequence. The local abundance of tracers (galaxies), at fixed proper time, is typically a function of the matter density field (and their spatial derivatives) within a finite region of size $R_* \sim$ few Mpc for most tracers. In most models of bias, the overdensities of the tracers and dark matter are understood as smoothed on some large-scale R_L so that they can be interpreted as a counts-in-cells relation. However, no additional smoothing scale R_L should enter in the final value of observables, e.g. the correlation functions on some scale r. This is because the smoothed scale R_L is not physical, it is just a tool for an effective description and an arbitrary ultra-violet cut-off [30].

The symmetries at our disposal provide a simple and straightforward way to show that the galaxy correlation functions do not depend on the smoothing scale R_L. Indeed, suppose we work in Fourier space and that we change the smoothing scale R_L by an infinitesimal amount δR_L. Correspondingly, the Fourier transformed window function will be

$$W\big[q(R_L+\delta R_L)\big] \simeq W(qR_L) + qW'(qR_L)\delta R_L \simeq W(qR_L)e^{qW'(qR_L)/W(qR_L)\delta R_L}, \tag{3.17}$$

where the prime stands for the differentiation with respect to the variable qR_L. We can perform now an infinitesimal coordinate transformation $\vec{x}' = \vec{x} + \vec{n}(\tau)$. According to the relation (3.16), both tracers and dark matter overdensities will transform in momentum space as

$$\delta'_{\vec{q},R_L} = \delta_{\vec{q},R_L} e^{i\vec{q}\cdot\vec{n}(\tau)} = \delta_{\vec{q}} W(qR_L) e^{i\vec{q}\cdot\vec{n}(\tau)} \tag{3.18}$$

and therefore

$$\delta'_{\vec{q},R_L+\delta R_L} = \delta_{\vec{q}} W\big[q(R_L+\delta R_L)\big] e^{i\vec{q}\cdot\vec{n}(\tau)} = \delta_{\vec{q},R_L} e^{qW'(qR_L)/W(qR_L)\delta R_L} e^{i\vec{q}\cdot\vec{n}(\tau)}. \tag{3.19}$$

We see that if we choose the infinitesimal vector $\vec{n}(\tau)$ to be

$$\vec{n}(\tau) = i\frac{\vec{q}}{q}\frac{W'(qR_L)}{W(qR_L)}\delta R_L, \tag{3.20}$$

we can compensate the infinitesimal change of the smoothing radius R_L and obtain that

$$\delta'_{\vec{q},R_L+\delta R_L} = \delta_{\vec{q},R_L}. \tag{3.21}$$

Since the correlators in the old and the new coordinate system have to be the same, we conclude that the dependence on the smoothing radius R_L drops off. Physically, this is due to the fact that changing the large-scale smoothing radius by some amount amounts to include (or exclude) more momentum modes into the smoothed overdensity. This addition (or subtraction) of momentum modes can be compensated by going to a coordinate system where these long wavelength modes have been removed (or added). This argument holds in all epochs, included the Λ-dominated epoch. During the matter-dominated epoch we have another tool to reach the same conclusion: the Lifshitz symmetry. Indeed, the change in the smoothing scale R_L can be compensated by a scaling transformation $\vec{x}' = \lambda\vec{x}$, or $\vec{q}' = \vec{q}/\lambda$. In such a case we have

$$\delta'_{\vec{q},R_L+\delta R_L} = \delta_{\vec{q}/\lambda} W\big[q/\lambda(R_L+\delta R_L)\big]. \tag{3.22}$$

If we choose $\lambda = \lambda_{R_L} = (1+\delta R_L/R_L)$, we obtain

$$\delta'_{\vec{q},R_L+\delta R_L} = \delta_{\vec{q}/\lambda_{R_L},R_L}, \tag{3.23}$$

and again we conclude that the smoothing scale dependence drops off when correlators are considered.

3.2. Galaxy bispectrum consistency relation at tree-level

Since the bias model (3.6) respects the symmetries (2.7)–(2.13), the three-point function of galaxies computed in this model should satisfy the consistency relation. In the next two subsections we explicitly verify that this is the case in perturbation theory at the tree and one-loop levels. Let us start with the tree-level case. The equal time DM-galaxy cross-correlation at second order in perturbation theory is

$$\big\langle \delta^{(1)}(\vec{k},\tau)\delta_{\rm g}^{(1)}(-\vec{k},\tau)\big\rangle' = b_1(\tau)P_{\delta_{\rm lin}}(k,\tau), \tag{3.24}$$

while the unequal time power spectrum is

$$\big\langle \delta_{\rm g}^{(1)}(\vec{k},\tau_1)\delta_{\rm g}^{(1)}(-\vec{k},\tau_2)\big\rangle' = b_1(\tau_1)b_1(\tau_2)\big\langle \delta^{(1)}(\vec{k},\tau_1)\delta^{(1)}(-\vec{k},\tau_2)\big\rangle'. \tag{3.25}$$

The bispectrum of the galaxies at fourth order for unequal times is

$$\begin{aligned}
&\big\langle \delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\delta_{\rm g}(\vec{k}_2,\tau_2)\big\rangle'\big|_{\delta^{(4)}} \\
&\quad = b_1(\tau_1)b_1(\tau_2)\big\langle \delta^{(1)}(\vec{k}_1,\tau_1)\delta^{(1)}(-\vec{k}_1,\tau)\big\rangle'\big\langle \delta^{(1)}(\vec{k}_2,\tau_2)\delta^{(1)}(-\vec{k}_2,\tau)\big\rangle' \\
&\qquad \times \big[2b_1(\tau)F_S^{(2)}(\vec{k}_1,\vec{k}_2) + b_2(\tau) + c_{s^2}(\tau)S(\vec{k}_1,\vec{k}_2)\big] \\
&\qquad + \text{cyclic permutations of } (\tau,\vec{q}),\ (\tau_1,\vec{k}_1) \text{ and } (\tau_2,\vec{k}_2),
\end{aligned} \tag{3.26}$$

where

$$F_S^{(2)}(\vec{k}_1,\vec{k}_2)=\left[\frac{5}{7}+\frac{1}{2}(\vec{k}_1\cdot\vec{k}_2)\frac{k_1^2+k_2^2}{k_1^2k_2^2}+\frac{2}{7}\frac{(\vec{k}_1\cdot\vec{k}_2)^2}{k_1^2k_2^2}\right],$$
$$S(\vec{k}_1,\vec{k}_2)=-\frac{1}{3}+\frac{(\vec{k}_1\cdot\vec{k}_2)^2}{k_1^2k_2^2}. \tag{3.27}$$

In the squeezed limit $q\to 0$, $\vec{k}_1\simeq-\vec{k}_2$ we find

$$\begin{aligned}&\big\langle\delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\delta_{\rm g}(\vec{k}_2,\tau_2)\big\rangle'_{q\to 0}\big|_{\delta^{(4)}}\\
&=b_1(\tau)b_1(\tau_1)b_1(\tau_2)\frac{\vec{q}\cdot\vec{k}_1}{q^2}\big\langle\delta^{(1)}(\vec{k}_1,\tau_1)\delta^{(1)}(-\vec{k}_1,\tau_2)\big\rangle'\\
&\quad\times\big(\big\langle\delta^{(1)}(\vec{q},\tau)\delta^{(1)}(-\vec{q},\tau_2)\big\rangle'-\big\langle\delta^{(1)}(\vec{q},\tau)\delta^{(1)}(-\vec{q},\tau_1)\big\rangle'\big)\\
&=b_1(\tau)b_1(\tau_1)b_1(\tau_2)\frac{\vec{q}\cdot\vec{k}_1}{q^2}\big\langle\delta^{(1)}(\vec{k}_1,\tau_1)\delta^{(1)}(-\vec{k}_1,\tau_2)\big\rangle'P_{\delta_{\rm lin}}(q,\tau)\left(\frac{D(\tau_2)}{D(\tau)}-\frac{D(\tau_1)}{D(\tau)}\right)\\
&=-\big\langle\delta_{\rm g}^{(1)}(\vec{q},\tau)\delta^{(1)}(\vec{q},\tau)\big\rangle'\left(\frac{D(\tau_1)}{D(\tau)}-\frac{D(\tau_2)}{D(\tau)}\right)\frac{\vec{q}\cdot\vec{k}_1}{q^2}\big\langle\delta_{\rm g}^{(1)}(\vec{k}_1,\tau_1)\delta_{\rm g}^{(1)}(\vec{k}_2,\tau_2)\big\rangle'.\end{aligned} \tag{3.28}$$

We observe that the consistency relation is trivially satisfied at linear order. One should note that non-local terms are sub-leading. We shall therefore ignore them in the one-loop computation and consider only the local-bias model in the following.

3.3. Galaxy bispectrum consistency relation at one-loop

The check the consistency relation at one-loop, or more precisely at order 6 in perturbation theory, we have to evaluate the following expression

$$\begin{aligned}&\big\langle\delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\delta_{\rm g}(\vec{k}_2,\tau_2)\big\rangle'\big|_{\delta^{(6)}}\\
&=\frac{\vec{q}\cdot\vec{k}_1}{q^2}\left(\frac{D(\tau_2)}{D(\tau)}-\frac{D(\tau_1)}{D(\tau)}\right)\big[\big\langle\delta^{(1)}(\vec{q},\tau)\delta_{\rm g}(-\vec{q},\tau)\big\rangle'\big\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\delta_{\rm g}(\vec{k}_2,\tau_2)\big\rangle'\big]_{\delta^{(6)}}.\end{aligned} \tag{3.29}$$

We first consider the right-hand side where one should be careful when expanding the square parenthesis. Indeed, even when δ is in the linear regime, $\delta_{\rm g}$ might be non-linear and higher order corrections to $\delta_{\rm g}$ have to be taken into account. The square parenthesis at order 4 in perturbation theory is therefore

$$\begin{aligned}&\big[\big\langle\delta^{(1)}(\vec{q},\tau)\delta_{\rm g}(-\vec{q},\tau)\big\rangle'\big\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\delta_{\rm g}(\vec{k}_2,\tau_2)\big\rangle'\big]_{\delta^{(6)}}\\
&=\big\langle\delta^{(1)}(\vec{q},\tau)\delta_{\rm g}^{(1)}(-\vec{q},\tau)\big\rangle'\big\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\delta_{\rm g}(\vec{k}_2,\tau_2)\big\rangle'\big|_{\delta^{(4)}}\\
&\quad+\big\langle\delta^{(1)}(\vec{q},\tau)\delta_{\rm g}^{(3)}(-\vec{q},\tau)\big\rangle'\big\langle\delta_{\rm g}^{(1)}(\vec{k}_1,\tau_1)\delta_{\rm g}^{(1)}(\vec{k}_2,\tau_2)\big\rangle',\end{aligned}$$

where $\delta_{\rm g}^{(3)}(\vec{q},\tau)$ is the third order contribution to $\delta_{\rm g}(\vec{q},\tau)$. The first term on the right-hand side can be written using the bias model as

$$\begin{aligned}&\big\langle\delta^{(1)}(\vec{q},\tau)\delta_{\rm g}^{(1)}(-\vec{q},\tau)\big\rangle'\big\langle\delta_{\rm g}(\vec{k}_1,\tau_1)\delta_{\rm g}(\vec{k}_2,\tau_2)\big\rangle'\big|_{\delta^{(4)}}\\
&=b_1(\tau)\big\langle\delta^{(1)}(\vec{q},\tau)\delta^{(1)}(-\vec{q},\tau)\big\rangle'\big(P_{\rm g}^{11}+P_{\rm g}^{12}+P_{\rm g}^{22}+P_{\rm g}^{13}\big),\end{aligned} \tag{3.30}$$

where

$$P_{\rm g}^{11} = b_1(\tau_1)b_1(\tau_2)\big\langle\delta(\vec{k}_1,\tau_1)\delta(\vec{k}_2,\tau_2)\big\rangle'\big|_{\delta^{(4)}}, \tag{3.31}$$

$$P_{\rm g}^{12} = \frac{1}{2}b_1(\tau_1)b_2(\tau_2)\int {\rm d}^3p\,\big\langle\delta(\vec{k}_1,\tau_1)\delta(\vec{p},\tau_2)\delta(\vec{k}_2-\vec{p},\tau_2)\big\rangle'\big|_{\delta^{(4)}} + (\vec{k}_1,\tau_1)\leftrightarrow(\vec{k}_2,\tau_2), \tag{3.32}$$

$$P_{\rm g}^{22} = \frac{b_2(\tau_1)b_2(\tau_2)}{2}\int {\rm d}^3p\,\big\langle\delta^{(1)}(\vec{p},\tau_1)\delta^{(1)}(-\vec{p},\tau_2)\big\rangle' \times\big\langle\delta^{(1)}(\vec{k}_1-\vec{p},\tau_1)\delta^{(1)}(-\vec{k}_1+\vec{p},\tau_2)\big\rangle', \tag{3.33}$$

$$P_{\rm g}^{13} = \frac{b_1(\tau_1)b_3(\tau_2)}{2}\big\langle\delta^{(1)}(\vec{k}_1,\tau_1)\delta^{(1)}(\vec{k}_2,\tau_2)\big\rangle'\sigma_L^2(\tau_2) + (\vec{k}_1,\tau_1)\leftrightarrow(\vec{k}_2,\tau_2), \tag{3.34}$$

while the second term is

$$\big\langle\delta^{(1)}(\vec{q},\tau)\delta_{\rm g}^{(3)}(-\vec{q},\tau)\big\rangle'\big\langle\delta_{\rm g}^{(1)}(\vec{k}_1,\tau_1)\delta_{\rm g}^{(1)}(\vec{k}_2,\tau_2)\big\rangle' = \frac{1}{2}b_3(\tau)b_1(\tau_1)b_1(\tau_2)\sigma_L^2(\tau)\big\langle\delta^{(1)}(\vec{q},\tau)\delta^{(1)}(-\vec{q},\tau)\big\rangle'\big\langle\delta^{(1)}(\vec{k}_1,\tau_1)\delta^{(1)}(\vec{k}_2,\tau_2)\big\rangle', \tag{3.35}$$

where we defined the linear variance

$$\sigma_L^2(\tau) \equiv \int {\rm d}^3p\, P_{\delta_{\rm lin}}(p,\tau). \tag{3.36}$$

Let us now compute the left-hand side of Eq. (3.29) with the help the expressions one can find in Ref. [34] and check that the equality is satisfied. The unequal-time bispectrum $\langle\delta_{\rm g}(\vec{q},\tau)\delta_{\rm g}(\vec{k}_1,\tau_1)\delta_{\rm g}(\vec{k}_2,\tau_2)\rangle'$ is composed by several terms which, for compactness, we will denote analogously to what done in Ref. [34] by the notation

$$\delta^D(\vec{q}+\vec{k}_1+\vec{k}_2)B^{ijk}_{{\rm g},q\to 0} \equiv \lim_{q\to 0}\Bigg[\frac{b_i(\tau)b_j(\tau_1)b_k(\tau_2)}{i!j!k!}\big\langle\delta^i(\vec{q},\tau)\delta^j(\vec{k}_1,\tau_1)\delta^k(\vec{k}_2,\tau_2)\big\rangle + \text{permutations } (\tau,\vec{q}),(\tau_1,\vec{k}_1),(\tau_2,\vec{k}_2)\Bigg]. \tag{3.37}$$

In the following, we compute each term identifying the ones which behave at least $\mathcal{O}(q^{-1}P_{\delta_{\rm lin}}(q))$ as $q\to 0$.

• The first term is

$$\begin{aligned} B^{111}_{{\rm g},q\to 0} &= b_1(\tau)b_1(\tau_1)b_1(\tau_2)\big\langle\delta(\vec{q},\tau)\delta(\vec{k}_1,\tau_1)\delta(\vec{k}_2,\tau_2)\big\rangle'_{q\to 0}\big|_{\delta^{(6)}} \\ &= b_1(\tau)b_1(\tau_1)b_1(\tau_2)\Bigg[P_{\delta_{\rm lin}}(q,\tau)\frac{\vec{q}\cdot\vec{k}_1}{q^2}\left(\frac{D(\tau_2)}{D(\tau)}-\frac{D(\tau_1)}{D(\tau)}\right) \\ &\quad\times\big\langle\delta(\vec{k}_1,\tau_1)\delta(\vec{k}_2,\tau_2)\big\rangle'\big|_{\delta^{(4)}}\Bigg], \end{aligned} \tag{3.38}$$

where we used the consistency relation for matter. This is exactly the term proportional to $P_{\rm g}^{11}$ in Eq. (3.31) in the right-hand side of the consistency relation.

• We express the trispectrum in the integral of the following term using the consistency relation

$$
\begin{aligned}
B^{112,II}_{\mathrm{g},q\to 0} &= \frac{1}{2} b_1(\tau) b_1(\tau_1) b_2(\tau_2) \int \mathrm{d}^3 p \left\langle \delta(\vec{q},\tau)\delta(\vec{k}_1,\tau_1)\delta(\vec{p},\tau_2)\delta(\vec{k}_2-\vec{p},\tau_2)\right\rangle'_{q\to 0}\Big|_{\delta^{(6)}} \\
&\quad + 2\ \text{perm.} \\
&= \frac{1}{2} b_1(\tau) b_1(\tau_1) b_2(\tau_2) \int \mathrm{d}^3 p \left\langle \delta(\vec{q},\tau)\delta(\vec{k}_1,\tau_1)\delta(\vec{p},\tau_2)\delta(\vec{k}_2-\vec{p},\tau_2)\right\rangle'_{q\to 0}\Big|_{\delta^{(6)}} \\
&\quad + (\tau_1,\vec{k}_1) \leftrightarrow (\tau_2,\vec{k}_2) \\
&= -P_{\delta_{\rm lin}}(q,\tau) \int \mathrm{d}^3 p \left[\frac{\vec{q}\cdot\vec{k}_1}{q^2}\frac{D(\tau_1)}{D(\tau)} + \frac{\vec{q}\cdot\vec{p}}{q^2}\frac{D(\tau_2)}{D(\tau)} + \frac{\vec{q}\cdot(\vec{k}_2-\vec{p})}{q^2}\frac{D(\tau_2)}{D(\tau)} \right] \\
&\quad \times \left\langle \delta(\vec{k}_1,\tau_1)\delta(\vec{p},\tau)\delta(\vec{k}_2-\vec{p},\tau_2)\right\rangle'\Big|_{\delta^{(4)}} + (\tau_1,\vec{k}_1) \leftrightarrow (\tau_2,\vec{k}_2) \\
&= -P_{\delta_{\rm lin}}(q,\tau) \frac{\vec{q}\cdot\vec{k}_1}{q^2} \left[\frac{D(\tau_1)}{D(\tau)} - \frac{D(\tau_2)}{D(\tau)} \right] \int \mathrm{d}^3 p \left\langle \delta(\vec{k}_1,\tau_1)\delta(\vec{p},\tau)\delta(\vec{k}_2-\vec{p},\tau_2)\right\rangle'\Big|_{\delta^{(4)}} \\
&\quad + (\tau_1,\vec{k}_1) \leftrightarrow (\tau_2,\vec{k}_2).
\end{aligned}
\tag{3.39}
$$

This is equal to the term proportional to P^{12}_{g} in Eq. (3.32). In the second line we ignored the permutation containing a bispectrum not in the squeezed limit.

- The following contribution reproduces the term proportional to P^{22}_{g} in Eq. (3.33).

$$
\begin{aligned}
B^{122,II}_{\mathrm{g},q\to 0} &= b_1(\tau) b_2(\tau_1) b_2(\tau_2) \int \mathrm{d}^3 p \left\langle \delta(\vec{q},\tau)\delta(\vec{p},\tau_1)\delta(-\vec{q}-\vec{p},\tau_2)\right\rangle'_{q\to 0}\Big|_{\delta^{(4)}} \\
&\quad \times \left\langle \delta^{(1)}(\vec{k}_1-\vec{p},\tau_1)\delta^{(1)}(-\vec{k}_1+\vec{p},\tau_2)\right\rangle' +\ 2\ \text{perm.} \\
&= b_1(\tau) b_2(\tau_1) b_2(\tau_2) \int \mathrm{d}^3 p \left\langle \delta(\vec{q},\tau)\delta(\vec{p},\tau_1)\delta(\vec{q}+\vec{p},\tau_2)\right\rangle'_{q\to 0}\Big|_{\delta^{(4)}} \\
&\quad \times \left\langle \delta^{(1)}(\vec{k}_1-\vec{p},\tau_1)\delta^{(1)}(\vec{k}_1-\vec{p},\tau_2)\right\rangle' \\
&= -b_1(\tau) b_2(\tau_1) b_2(\tau_2) P_{\delta_{\rm lin}}(q,\tau) \left[\frac{D(\tau_1)}{D(\tau)} - \frac{D(\tau_2)}{D(\tau)} \right] \\
&\quad \times \int \mathrm{d}^3 p \, \frac{\vec{q}\cdot(\vec{k}_1-\vec{p})}{q^2} \left\langle \delta^{(1)}(\vec{k}_1-\vec{p},\tau_1)\delta^{(1)}(-\vec{k}_1+\vec{p},\tau_2)\right\rangle' \\
&\quad \times \left\langle \delta^{(1)}(\vec{p},\tau_1)\delta^{(1)}(-\vec{p},\tau_2)\right\rangle' \\
&= -\frac{1}{2} b_1(\tau) b_2(\tau_1) b_2(\tau_2) P_{\delta_{\rm lin}}(q,\tau) \left[\frac{D(\tau_1)}{D(\tau)} - \frac{D(\tau_2)}{D(\tau)} \right] \frac{\vec{q}\cdot\vec{k}_1}{q^2} \\
&\quad \times \int \mathrm{d}^3 p \left\langle \delta^{(1)}(\vec{k}_1-\vec{p},\tau_1)\delta^{(1)}(-\vec{k}_1+\vec{p},\tau_2)\right\rangle' \left\langle \delta^{(1)}(\vec{p},\tau_1)\delta^{(1)}(-\vec{p},\tau_2)\right\rangle'.
\end{aligned}
\tag{3.40}
$$

In the second equality we kept the only permutation enhanced in the squeezed limit and in the third we used the consistency relation for matter. Finally, we used the fact that

$$
\begin{aligned}
&\int \mathrm{d}^3 p \, \frac{\vec{q}\cdot\vec{p}}{q^2} \left\langle \delta^{(1)}(\vec{k}_1-\vec{p},\tau_1)\delta^{(1)}(-\vec{k}_1+\vec{p},\tau_2)\right\rangle' \left\langle \delta^{(1)}(\vec{p},\tau_1)\delta^{(1)}(-\vec{p},\tau_2)\right\rangle' \\
&\quad = \frac{1}{2} \int \mathrm{d}^3 p \, \frac{\vec{q}\cdot\vec{k}_1}{q^2} \left\langle \delta^{(1)}(\vec{k}_1-\vec{p},\tau_1)\delta^{(1)}(-\vec{k}_1+\vec{p},\tau_2)\right\rangle' \left\langle \delta^{(1)}(\vec{p},\tau_1)\delta^{(1)}(-\vec{p},\tau_2)\right\rangle',
\end{aligned}
\tag{3.41}
$$

which can be deduced simply by doing the shift $\vec{p} \to \vec{k}_1 - \vec{p}$.

• The term below is enhanced in the squeezed limit as it contains a bispectrum at unequal times. It reproduces the term proportional to $P_{\rm g}^{13}$ in Eq. (3.34) together with the term in Eq. (3.35)

$$
\begin{aligned}
B^{113,II}_{{\rm g},q\to 0} &= \left[\frac{1}{2} b_1(\tau) b_1(\tau_1) b_3(\tau_2)\sigma_L^2(\tau_2) + 2\ \text{perm.}\right]\big\langle \delta(\vec{q},\tau)\delta(\vec{k}_1,\tau_1)\delta(\vec{k}_2,\tau_2)\big\rangle'_{q\to 0}\big|_{\delta^{(4)}} \\
&= \left[\frac{1}{2} b_1(\tau) b_1(\tau_1) b_3(\tau_2)\sigma_L^2(\tau_2) + 2\ \text{perm.}\right] \\
&\quad \times \frac{\vec{q}\cdot\vec{k}_1}{q^2} P_{\delta_{\rm lin}}(q,\tau)\left(\frac{D(\tau_2)}{D(\tau)} - \frac{D(\tau_1)}{D(\tau)}\right)\big\langle \delta^{(1)}(\vec{k}_1,\tau_1)\delta^{(1)}(\vec{k}_2,\tau_2)\big\rangle'.
\end{aligned} \tag{3.42}
$$

• The term

$$
B^{112,I}_{{\rm g},q\to 0} = b_1^2 b_2\big[P(q)P(k_1)\big]_{\delta^{(6)}} + 2\ \text{perm.} = \mathcal{O}\big(P_{\delta_{\rm lin}}(q)\big) \tag{3.43}
$$

is not dominant because the $\mathcal{O}(\delta^{(4)})$ corrections to $P(q)$ are at most $\mathcal{O}(P_{\delta_{\rm lin}}(q))$ when $q \to 0$.

• The following terms are not relevant because they involve either terms that are proportional to the non-squeezed bispectrum, which makes them at most $\mathcal{O}(P_{\delta_{\rm lin}}(q))$, or terms containing the bispectrum in the squeezed limit at equal times, which vanish due to the consistency relation. B denotes the bispectrum of matter

$$
B^{122,I}_{{\rm g},q\to 0} = \frac{b_1 b_2^2}{2} P_{\delta_{\rm lin}}(k_1)\int {\rm d}^3p\, B|_{\delta^{(4)}}\big(k_2, p, |\vec{k}_2 - \vec{p}|\big) + 5\ \text{perm.} = \mathcal{O}\big(P_{\delta_{\rm lin}}(q)\big), \tag{3.44}
$$

$$
B^{113,I}_{{\rm g},q\to 0} = \frac{b_1^2 b_3}{2} P_{\delta_{\rm lin}}(k_1)\int {\rm d}^3p\, B|_{\delta^{(4)}}\big(k_1, p, |\vec{k}_1 - \vec{p}|\big) + 5\ \text{perm.} = \mathcal{O}\big(P_{\delta_{\rm lin}}(q)\big). \tag{3.45}
$$

• The following terms are not enhanced in the squeezed limit as they are just products of linear power spectra at this order

$$
B^{222}_{{\rm g},q\to 0} = \frac{b_2^3}{2}\int {\rm d}^3p\, P_{\delta_{\rm lin}}(p) P_{\delta_{\rm lin}}\big(|\vec{q}+\vec{p}|\big) P_{\delta_{\rm lin}}\big(|\vec{k}_1 - \vec{p}|\big) = \mathcal{O}(1), \tag{3.46}
$$

$$
B^{123,I}_{{\rm g},q\to 0} = \frac{b_1 b_2 b_3}{2} P_{\delta_{\rm lin}}(q)\int {\rm d}^3p\, P_{\delta_{\rm lin}}\big(|\vec{k}_1 - \vec{p}|\big) P_{\delta_{\rm lin}}(p) + 5\ \text{perm.} = \mathcal{O}\big(P_{\delta_{\rm lin}}(q)\big), \tag{3.47}
$$

$$
B^{123,II}_{{\rm g},q\to 0} = b_1 b_2 b_3 P_{\delta_{\rm lin}}(q) P_{\delta_{\rm lin}}(k_1)\sigma_L^2 + 2\ \text{perm.} = \mathcal{O}\big(P_{\delta_{\rm lin}}(q)\big), \tag{3.48}
$$

$$
B^{114,I}_{{\rm g},q\to 0} = \frac{b_1^2 b_4}{2} P_{\delta_{\rm lin}}(q) P_{\delta_{\rm lin}}(k_1)\sigma_L^2 + 2\ \text{perm.} = \mathcal{O}\big(P_{\delta_{\rm lin}}(q)\big). \tag{3.49}
$$

• Finally, the two- and three-loops corrections ignored in Ref. [34] are at most constant in the squeezed limit such that our result is fully correct at sixth order. Overall, we conclude that the galaxy consistency relation is satisfied at tree- and one-loop level.

4. Consistency relation of galaxy correlation functions in redshift space

Let us discuss now discuss how the galaxy consistent relations are modified when going from real space to redshift space where experiments are performed. The mapping from real-space position $\vec{x}$ to redshift space $\vec{s}$ is given by [32]

$$
\vec{s} = \vec{x} + \frac{1}{\mathcal{H}}(\vec{v}_{\rm g}\cdot\hat{x})\hat{x}, \tag{4.1}
$$

and the density field in redshift space is obtained by imposing mass conservation

$$\big[1+\delta_{\rm g}(\vec{s})\big]\,{\rm d}^3 s=\big[1+\delta_{\rm g}(\vec{x})\big]\,{\rm d}^3 x. \tag{4.2}$$

In Fourier space the condition (4.2) reads

$$\delta_{\rm D}(\vec{k})+\delta_{{\rm g},s}(\vec{k})=\int\frac{{\rm d}^3 x}{(2\pi)^3}\,e^{-i\vec{k}\cdot\vec{x}}\,e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}\,(\vec{k}\cdot\hat{x})/\mathcal{H}}\big[1+\delta_{\rm g}(\vec{x})\big]. \tag{4.3}$$

By performing a spatial coordinate transformation $\vec{x}\to\vec{x}\,'=\vec{x}+\vec{n}(\tau)$ we know that, if $\delta_{\rm g}(\vec{x},\tau)$ and $\vec{v}(\vec{x},\tau)$ satisfy the fluid equations, then $\delta'_{\rm g}(\vec{x},\tau)=\delta_{\rm g}(\vec{x}\,',\tau)$ and $\vec{v}\,'(\vec{x},\tau)=\vec{v}(\vec{x}\,',\tau)-\dot{\vec{n}}$ do as well. This implies that for the new solution we have

$$\begin{aligned}
\delta_{\rm D}(\vec{k})+\delta'_{{\rm g},s}(\vec{k})&=\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}\,'_{\rm g}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}\big[1+\delta'_{\rm g}(\vec{x})\big]\\
&=\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\rm g}(\vec{x}\,')\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}e^{i\dot{\vec{n}}\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}\big[1+\delta_{\rm g}\big(\vec{x}\,'\big)\big]\\
&\simeq\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}\,e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}e^{-i[(\vec{n}\cdot\vec{\nabla})\vec{v}_{\rm g}]\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}e^{i\dot{\vec{n}}\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}\\
&\quad\times\big[1+\delta_{\rm g}(\vec{x})+(\vec{n}\cdot\vec{\nabla})\delta_{\rm g}(\vec{x})\big].
\end{aligned} \tag{4.4}$$

This expression is exact. Expanding for small $\vec{n}(\tau)$, we get

$$\begin{aligned}
\delta_{\rm D}(\vec{k})+\delta'_{{\rm g},s}(\vec{k})&\simeq\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}\big[1+\delta_{\rm g}(\vec{x})\big]\\
&\quad+\frac{1}{\mathcal{H}}\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}(\vec{k}\cdot\hat{x})\\
&\quad\times\big\{-i\big[(\vec{n}\cdot\vec{\nabla})\vec{v}_{\rm g}(\vec{x})\big]\cdot\hat{x}+i(\dot{\vec{n}}\cdot\hat{x})\big\}\big[1+\delta_{\rm g}(\vec{x})\big]\\
&\quad+\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}(\vec{n}\cdot\vec{\nabla})\delta_{\rm g}(\vec{x})\\
&=\delta_{\rm D}(\vec{k})+\delta_{{\rm g},s}(\vec{k})+\frac{1}{\mathcal{H}}\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}\,(\vec{k}\cdot\hat{x})/\mathcal{H}}(\vec{k}\cdot\hat{x})\\
&\quad\times\big\{-i\big[(\vec{n}\cdot\vec{\nabla})\vec{v}_{\rm g}(\vec{x})\big]\cdot\hat{x}+i(\dot{\vec{n}}\cdot\hat{x})\big\}\big[1+\delta_{\rm g}(\vec{x})\big]\\
&\quad+\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}\,(\vec{k}\cdot\hat{x})/\mathcal{H}}(\vec{n}\cdot\vec{\nabla})\big[1+\delta_{\rm g}(\vec{x})\big].
\end{aligned} \tag{4.5}$$

If we start from this expression, upon integrating by parts we find

$$\begin{aligned}
\delta_{\rm D}(\vec{k})+\delta'_{{\rm g},s}(\vec{k})&=\delta_{\rm D}(\vec{k})+\delta_{{\rm g},s}(\vec{k})+\frac{i}{\mathcal{H}}\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}\,(\vec{k}\cdot\hat{x})/\mathcal{H}}(\vec{k}\cdot\hat{x})\\
&\quad\times\big\{-\big[(\vec{n}\cdot\vec{\nabla})\vec{v}_{\rm g}(\vec{x})\big]\cdot\hat{x}+(\dot{\vec{n}}\cdot\hat{x})\big\}\big[1+\delta_{\rm g}(\vec{x})\big]\\
&\quad+\frac{i}{\mathcal{H}}\int\frac{{\rm d}^3 x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}(\vec{n}\cdot\vec{\nabla})\\
&\quad\times\big\{\vec{v}_{\rm g}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})\big\}\big[1+\delta_{\rm g}(\vec{x})\big]+i(\vec{k}\cdot\vec{n})\delta_{{\rm g},s}(\vec{k}).
\end{aligned} \tag{4.6}$$

This gives

$$\begin{aligned}\delta_{\mathrm{D}}(\vec{k})+\delta'_{\mathrm{g},s}(\vec{k})=&\ \delta_{\mathrm{D}}(\vec{k})+\delta_{\mathrm{g},s}(\vec{k})+i(\vec{k}\cdot\vec{n})\delta_{\mathrm{g},s}(\vec{k})\\&+\frac{i}{\mathcal{H}}\int\frac{\mathrm{d}^3x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\mathrm{g}}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}(\vec{k}\cdot\hat{x})\\&\times\left\{\left[(\vec{n}\cdot\vec{\nabla})\hat{x}\right]\cdot\vec{v}_{\mathrm{g}}(\vec{x})+(\dot{\vec{n}}\cdot\hat{x})\right\}\left[1+\delta_{\mathrm{g}}(\vec{x})\right]\\&+\frac{i}{\mathcal{H}}\int\frac{\mathrm{d}^3x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\mathrm{g}}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}\\&\times\left[\vec{v}_{\mathrm{g}}(\vec{x})\cdot\hat{x}\right]\left[1+\delta_{\mathrm{g}}(\vec{x})\right](\vec{n}\cdot\vec{\nabla})(\vec{k}\cdot\hat{x}).\end{aligned}\tag{4.7}$$

At this point we can use the distant observer approximation, that is take the direction of the vector $\vec{x}$ fixed, since it varies little from galaxy to galaxy: galaxies are relatively close to each other on the plane orthogonal to the line-of-sight. This amounts to taking $\vec{\nabla}\hat{x}\simeq\vec{0}$ and we finally obtain

$$\begin{aligned}\delta_{\mathrm{D}}(\vec{k})+\delta'_{\mathrm{g},s}(\vec{k})=&\ \delta_{\mathrm{D}}(\vec{k})+\delta_{\mathrm{g},s}(\vec{k})+i(\vec{k}\cdot\vec{n})\delta_{\mathrm{g},s}(\vec{k})\\&+\frac{i}{\mathcal{H}}\int\frac{\mathrm{d}^3x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\mathrm{g}}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}(\vec{k}\cdot\hat{x})(\dot{\vec{n}}\cdot\hat{x})\left[1+\delta_{\mathrm{g}}(\vec{x})\right].\end{aligned}\tag{4.8}$$

Note that here the first line corresponds to the field transformation that gives rise to the consistency relation, which in redshift space will contain new terms induced by the second line of this expression. Using the explicit expression for $\dot{\vec{n}}$

$$\dot{\vec{n}}(\tau)=-\vec{v}_L(\tau)=-i\frac{\vec{q}}{q^2}\mathcal{H}f(\tau)\delta_L(\vec{q},\tau),$$

we obtain,

$$\begin{aligned}\delta_{\mathrm{D}}(\vec{k})+\delta'_{\mathrm{g},s}(\vec{k})=&\ \delta_{\mathrm{D}}(\vec{k})+\delta_{\mathrm{g},s}(\vec{k})+\frac{\vec{k}\cdot\vec{q}}{q^2}\delta(\vec{q})\delta_{\mathrm{g},s}(\vec{k})\\&+f\frac{k}{q}\delta(\vec{q})\int\frac{\mathrm{d}^3x}{(2\pi)^3}e^{-i\vec{k}\cdot\vec{x}}e^{-i\vec{v}_{\mathrm{g}}(\vec{x})\cdot\hat{x}(\vec{k}\cdot\hat{x})/\mathcal{H}}\mu_{\vec{k}}\mu_{\vec{q}}\left[1+\delta_{\mathrm{g}}(\vec{x})\right]\\=&\ \delta_{\mathrm{D}}(\vec{k})+\delta_{\mathrm{g},s}(\vec{k})+\frac{\vec{k}\cdot\vec{q}}{q^2}\delta(\vec{q})\delta_{\mathrm{g},s}(\vec{k})+f\frac{k}{q}\mu_{\vec{k}}\mu_{\vec{q}}\delta(\vec{q})\delta_{\mathrm{g},s}(\vec{k}),\end{aligned}\tag{4.9}$$

where $\mu_{\vec{k}}$ is the cosine between the vector $\hat{k}$ and $\hat{x}$, and we used the distant observer approximation to take the cosines out of the integral in the second equality. We therefore obtain that in redshift space the consistency relations reads

$$\boxed{\begin{aligned}&\frac{\langle\delta_{\mathrm{g},s}(\vec{q},\tau)\delta_{\mathrm{g},s}(\vec{k}_1,\tau_1)\cdots\delta_{\mathrm{g},s}(\vec{k}_n,\tau_n)\rangle'_{q\to0}}{\langle\delta_{\mathrm{g},s}(\vec{q},\tau_1)\cdots\delta_{\mathrm{g},s}(\vec{k}_n,\tau_n)\rangle'}\\&\quad=-\frac{P_{\mathrm{g},s}(q,\tau)}{b_1(\tau)+f(\tau)\mu^2_{\vec{q}}}\sum_{a=1}^{n}\frac{D(\tau_a)}{D(\tau)}\left(\frac{\vec{q}\cdot\vec{k}_a}{q^2}+f(\tau_a)\frac{k_a}{q}\mu_{\vec{q}}\mu_{\vec{k}_a}\right)\end{aligned}}\tag{4.10}$$

where we have used the linear relation [28]

$$\delta_{\mathrm{g},s}(\vec{q},\tau)=\left[b_1(\tau)+f(\tau)\mu^2_{\vec{q}}\right]\delta(\vec{q},\tau).\tag{4.11}$$

In particular, the consistency relation for the bispectrum in redshift space explicitly reads

$$\frac{\langle\delta_{g,s}(\vec{q},\tau)\delta_{g,s}(\vec{k}_1,\tau_1)\cdots\delta_{g,s}(\vec{k}_2,\tau_2)\rangle'_{q\to 0}}{\langle\delta_{g,s}(\vec{k}_1,\tau_1)\delta_{g,s}(\vec{k}_2,\tau_2)\rangle'}$$
$$= -\frac{P_{g,s}(q,\tau)}{b_1(\tau)+f(\tau)\mu^2_{\vec{q}}}\left[\frac{\vec{q}\cdot\vec{k}_1}{q^2}\left(\frac{D(\tau_1)}{D(\tau)}-\frac{D(\tau_2)}{D(\tau)}\right)\right.$$
$$\left. + \left(\frac{D(\tau_1)}{D(\tau)}f(\tau_1)-\frac{D(\tau_2)}{D(\tau)}f(\tau_2)\right)\frac{k_1}{q}\mu_{\vec{k}_1}\mu_{\vec{q}}\right]. \tag{4.12}$$

5. Consequences of the symmetries for the modified theories of gravity

Theories that (attempt to) explain the observed cosmic acceleration by modifying general relativity all introduce a new scalar degree of freedom that is active on large scales, but is screened on small scales to match experiments. All these theories introduce an extra light scalar field to modified gravity in the infrared. Typical examples are represented by the $f(R)$ theories [33], which are equivalent to classic scalar-tensor theories [35] and the screening effect takes place through the so-called chameleon mechanism [36], and by Galileon theories [37] where the extra degree of freedom is appropriately dressed through higher-derivative interactions which decouple it form short-scale physics in accordance with solar system tests.

The chameleon for example, has a potential such that it has long range forces outside of objects while it is massive in their interior. Therefore, the existence of such field is consistent with solar system and fifth force tests but still can modify gravity at large distances. In the following we will be interested in two kinds of objects (typically galaxies): those sitting in some high density environment which is itself screened and those residing in an environment where the density is at the cosmic mean or even lower (voids), where the objects can be unscreened. In the first region the chameleon-like field is stuck at the minimum of its potential, while in the second region it is excited. As a result, matter in the first region will follow geodesics, whereas matter in the second region will experience a non-gravitational force, due to scalar gradient, departing from the geodesic motion. The galaxy and dark matter consistency relations are based on a coordinate transformation[1] (in a matter-dominated period) [16,18]

$$\tau'=\tau, \qquad \vec{x}'=\vec{x}+\int^{\tau} d\eta\, \vec{v}_L(\eta)=\vec{x}+\frac{1}{6}\tau^2\vec{\nabla}\Phi_L, \tag{5.1}$$

we are basically removing the time-dependent, but homogeneous gravitational force via a change of coordinates. This corresponds to an homogeneous acceleration transformation which allows to go to a free-falling observer. Therefore, one expects a violation (or a spatial dependence) of the galaxy consistency relation in modified gravity models where the screening mechanism is in action. Note that the consistency relations do not rely on the Equivalence Principle in a strict sense as pointed out in [38] but only require that large scale overdensities satisfy the same equations of motion. We will refer to a deviation from this as a violation of the “large-scale EP”.

Let us implement the presence of the chameleon-like field in the energy–momentum conservation of the non-relativistic dark matter fluid

$$\nabla_\mu T^{\mu\nu}=f^\mu. \tag{5.2}$$

[1] Note that here the gradient of the long-wavelength mode $\vec{\nabla}\Phi_L$ is taken to be a constant vector in space, i.e. recall that we are doing an expansion on the space variation of Φ_L and keep only terms linear in its gradients.

One finds

$$\frac{\partial\rho(\vec{x},\tau)}{\partial\tau}+\vec{\nabla}\cdot\big[\rho(\vec{x},\tau)\vec{v}(\vec{x},\tau)\big]=0 \tag{5.3}$$

$$\frac{\partial\vec{v}(\vec{x},\tau)}{\partial\tau}+\mathcal{H}(\tau)\vec{v}(\vec{x},\tau)+\big[\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}\big]\vec{v}(\vec{x},\tau)=-\vec{\nabla}\Phi(\vec{x},\tau)-\frac{q}{\rho}\vec{\nabla}\varphi(\vec{x},\tau), \tag{5.4}$$

where we assumed that

$$f^i=-q\partial^i\varphi(\vec{x},\tau), \tag{5.5}$$

with $\varphi(\vec{x},\tau)$ the scalar field that has environmental couplings that causes a violation of the "large-scale EP" and q is the scalar charge density of the fluid. We follow the parametrization used in Ref. [39] and we assume $q=\epsilon\alpha\rho$, where α is a constant and ϵ is a parameter that describes the degree of screening ($\epsilon=0$ for screened objects and $\epsilon=1$ for unscreened ones). We should supplement the above equations with the Poisson equation for the gravitational potential $\Phi(\vec{x},\tau)$ and a corresponding equation for $\varphi(\vec{x},\tau)$ which we write as

$$\nabla^2\Phi(\vec{x},\tau)=4\pi Ga^2\rho(\vec{x},\tau), \tag{5.6}$$

$$\nabla^2\varphi(\vec{x},\tau)=\left(\frac{\partial V}{\partial\varphi}+8\pi G\alpha\rho(\vec{x},\tau)\right)a^2, \tag{5.7}$$

where $V(\varphi)$ is the scalar potential of the chameleon-like field. We may now consider perturbations $\rho(\vec{x},\tau)=\bar{\rho}(1+\delta(\vec{x},\tau))$ and $\varphi(\vec{x},\tau)=(\bar{\varphi}+\delta\varphi(\vec{x},\tau))$ around the corresponding background values $\bar{\rho}$ and $\bar{\varphi}$ and we find that these perturbations satisfy the equations

$$\frac{\partial\delta(\vec{x},\tau)}{\partial\tau}+\vec{\nabla}\cdot\big[\big(1+\delta(\vec{x},\tau)\big)\vec{v}(\vec{x},\tau)\big]=0, \tag{5.8}$$

$$\frac{\partial\vec{v}(\vec{x},\tau)}{\partial\tau}+\mathcal{H}(\tau)\vec{v}(\vec{x},\tau)+\big[\vec{v}(\vec{x},\tau)\cdot\vec{\nabla}\big]\vec{v}(\vec{x},\tau)=-\vec{\nabla}\Phi(\vec{x},\tau)-\epsilon\alpha\vec{\nabla}\delta\varphi(\vec{x},\tau), \tag{5.9}$$

$$\nabla^2\Phi(\vec{x},\tau)=4\pi Ga^2\bar{\rho}\delta(\vec{x},\tau), \tag{5.10}$$

$$\nabla^2\delta\varphi(\vec{x},\tau)=\big(m^2\delta\varphi(\vec{x},\tau)+8\pi G\alpha\bar{\rho}\delta(\vec{x},\tau)\big)a^2, \tag{5.11}$$

where

$$m^2(\vec{x},\tau)=\left.\frac{\partial^2V}{\partial\varphi^2}\right|_{\bar{\varphi}} \tag{5.12}$$

is the mass of the scalar field. Restricting ourselves to the matter-dominated case, it can be checked that Eqs. (5.8)–(5.11) are invariant under the transformations

$$\tau'=\tau,\qquad \vec{x}'=\vec{x}+\vec{n}(\tau), \tag{5.13}$$

$$\delta'(\vec{x},\tau)=\delta\big(\vec{x}',\tau'\big), \tag{5.14}$$

$$\vec{v}'(\vec{x},\tau)=\vec{v}\big(\vec{x}',\tau'\big)-\dot{\vec{n}}(\tau), \tag{5.15}$$

$$\delta\varphi'(\vec{x},\tau)=\delta\varphi\big(\vec{x}',\tau'\big), \tag{5.16}$$

$$\Phi'(\vec{x},\tau)=\Phi\big(\vec{x}',\tau'\big)-\big(\ddot{\vec{n}}(\tau)+\mathcal{H}(\tau)\dot{\vec{n}}(\tau)\big)\cdot\vec{x}. \tag{5.17}$$

As a result, it is still possible to remove a long wavelength mode for the velocity perturbation $\vec{v}_L(\tau,\vec{0})$ by properly choosing the vector $\vec{n}(\tau)$ in order. Indeed, in the linear regime in momentum space the dynamical equations are given by

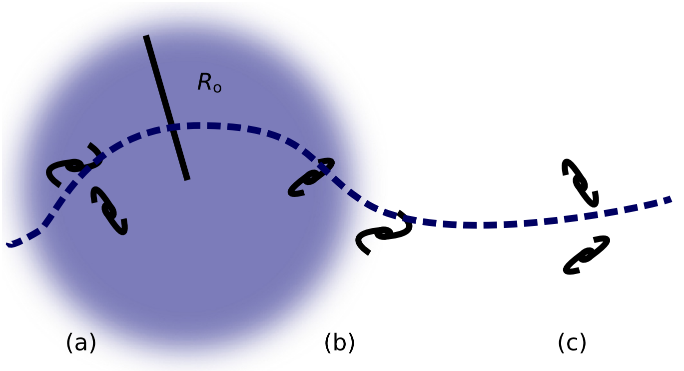

Fig. 1. Schematic representation of a large-scale spherical over-density of radius R_0 where the chameleon field is screened, and in the presence of a long-wavelength perturbation of the gravitational field (here represented by the dark blue dashed line). The consistency relation will be given by the correlation of the modulation of the power spectrum with the long-wavelength gravitational field. The case (a) corresponds to the case in which the galaxies are all in the screened region, Eq. (5.29), case (c) corresponds to the case in which all the galaxies are in the unscreened region, Eq. (5.30), and case (b) corresponds to the case in which there are both screened and unscreened galaxies, Eq. (5.31).

$$\frac{\partial \delta_L(\vec{q},\tau)}{\partial \tau} + i\vec{q}\cdot\vec{v}_L(\vec{q},\tau) = 0, \tag{5.18}$$

$$\frac{\partial \vec{v}_L(\vec{q},\tau)}{\partial \tau} + \mathcal{H}(\tau)\vec{v}_L(\vec{q},\tau) = -i\vec{q}\big(\Phi_L(\vec{q},\tau) + \alpha\epsilon\,\delta\varphi(\vec{q},\tau)\big), \tag{5.19}$$

$$q^2\Phi_L(\vec{q},\tau) = -\frac{3}{2}\mathcal{H}^2\Omega_{\rm m}\delta_L(\vec{q},\tau), \tag{5.20}$$

$$q^2\delta\varphi(\vec{q},\tau) = -\big(m^2\delta\varphi\big)(\vec{q},\tau)a^2 - 3\alpha\mathcal{H}^2\Omega_{\rm m}\delta_L(\vec{q},\tau), \tag{5.21}$$

where $(m^2\delta\varphi)(\vec{q},\tau)$ is the Fourier mode of $m^2(\vec{x},\tau)\delta\varphi(\vec{x},\tau)$. In particular consider the configurations shown in Fig. 1, in a region outside the spherical over-density of radius R_0 where the chameleon-like field is not screened and its mass may be neglected, one has

$$\delta\varphi(\vec{q},\tau) \simeq -\frac{3}{q^2}\mathcal{H}^2\Omega_{\rm m}\delta_L(\vec{q},\tau), \tag{5.22}$$

where the equation for the linear matter overdensity satisfies the equation[2]

$$\ddot{\delta}_L + \mathcal{H}(\tau)\dot{\delta}_L - 4\pi G a^2\big(1+2\alpha^2\epsilon\big)\bar{\rho}\delta_L = 0, \quad r \gtrsim R_0, \tag{5.23}$$

with solution $\delta_L^>(\tau) = D^>(\tau)/D^>(\tau_{\rm in})\delta(\tau_{\rm in})$ where $D^>(\tau)$ is the growth function for (5.23). On the contrary, in a screened region, where the field $\delta\varphi$ is massive enough so that φ is not excited, but fixed to some constant background value within a sphere of radius R_0. In such a case the equation for the overdensity is given by

$$\ddot{\delta}_L + \mathcal{H}(\tau)\dot{\delta}_L - 4\pi G a^2\bar{\rho}\delta_L = 0, \quad r \lesssim R_0 \tag{5.24}$$

and it is solved by $\delta_L^<(\tau) = D^<(\tau)/D^<(\tau_{\rm in})\delta_L(\tau_{\rm in})$. Therefore $\vec{v}_L$ will be different in the two regions $r \lesssim R_0$ and $r \gtrsim R_0$. As a result, two different vectors $\vec{n}$'s will be needed to generate (or remove) the long wave velocity perturbation, one for $r \lesssim R_0$ and the other $r \gtrsim R_0$: in the

[2] We use dots to denote derivatives with respect to conformal time.

presence of modified gravity exploiting the screening effect, it is not possible to find a spatially independent vector $\vec{n}(\tau)$ and the consistency relations must be violated for objects which are unscreened. This violation of the consistency relation should be attributed to the fact that the effect of the velocity long mode cannot be reabsorbed completely by a change of coordinates. As indicated in Fig. 1, galaxies residing in the same region (a) or (b) for example, fall differently even if they are of the same mass, due to the difference in their scalar charge.

The vector $\vec{n}(\tau)$ is chosen such as to have a free-falling frame, defined by

$$\ddot{\vec{n}} + \mathcal{H}\dot{\vec{n}} + \vec{\nabla}\Phi = 0. \tag{5.25}$$

The solution to this equation is

$$\dot{\vec{n}}(\tau) = -i\frac{\vec{q}}{a(\tau)}\int^{\tau} \mathrm{d}\eta\, a(\eta)\Phi(\vec{q},\eta) = i\frac{\vec{q}}{q^2}\frac{1}{a(\tau)}\int^{\tau} \mathrm{d}\eta\, a^3(\eta)4\pi G\bar{\rho}\delta_L(\vec{q},\eta). \tag{5.26}$$

Then, by using Eqs. (5.23) and (5.24), we find that the free-falling frame is specified by

$$\vec{n}(\tau) = i\frac{\vec{q}}{q^2}\delta_L^{<}(\vec{q},\tau), \quad r \lesssim R_0, \tag{5.27}$$

$$\vec{n}(\tau) = i\frac{\vec{q}}{q^2}\frac{\delta_L^{>}(\vec{q},\tau)}{1+2\alpha^2\epsilon}, \quad r \gtrsim R_0, \tag{5.28}$$

where $<$ and $>$ denote quantities respectively in the two regions $r \lesssim R_0$ and $r \gtrsim R_0$. This notation may look odd at first sight but we use it to indicate that linear modes grow differently in screened and unscreened regions as they may or may not feel the presence of the chameleon field. Thus, although the wavelength q^{-1} is larger than the two regions, the linear overdensity amplitude is different in these regions and for local observers, this linear mode is like a background average density field with different amplitudes in the two regions. This is also connected to the fact that there is no universal free falling frame according to (5.27), (5.28). Consider for example n-galaxies within a sphere of radius $R \gtrsim R_0$ much smaller than the long wavelength mode of size $\sim 1/q$ and centered at the origin of the coordinates. Then, if all points are at distances $r \lesssim R_0$, then the consistency relation for the n-point correlator is the one we already described

$$\begin{aligned}&\big\langle\delta_{\mathrm{g}}^{<}(\vec{q},\tau)\delta_{\mathrm{g}}^{<}(\vec{k}_1,\tau_1)\cdots\delta_{\mathrm{g}}^{<}(\vec{k}_n,\tau_n)\big\rangle'_{q\to 0}\\&\quad = -\sum_{a=1}^{n}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\big\langle\delta_{\mathrm{g},L}^{<}(\vec{q},\tau)\delta_L^{<}(\vec{q},\tau_a)\big\rangle\big\langle\delta_{\mathrm{g}}^{<}(\vec{k}_1,\tau_1)\cdots\delta_{\mathrm{g}}^{<}(\vec{k}_n,\tau_n)\big\rangle'.\end{aligned} \tag{5.29}$$

If instead all points are at $r \gtrsim R_0$, we will have in this case

$$\begin{aligned}&\big\langle\delta_{\mathrm{g}}^{>}(\vec{q},\tau)\delta_{\mathrm{g}}^{>}(\vec{k}_1,\tau_1)\cdots\delta_{\mathrm{g}}^{>}(\vec{k}_n,\tau_n)\big\rangle'_{q\to 0}\\&\quad = -\sum_{a=1}^{n}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\frac{\langle\delta_{\mathrm{g},L}^{>}(\vec{q},\tau)\delta_L^{>}(\vec{q},\tau_a)\rangle}{1+2\alpha^2\epsilon}\big\langle\delta_{\mathrm{g}}^{>}(\vec{k}_1,\tau_1)\cdots\delta_{\mathrm{g}}^{>}(\vec{k}_n,\tau_n)\big\rangle'.\end{aligned} \tag{5.30}$$

The case in which galaxies are both screened and unscreened is more complex,[3] however we expect a violation of the consistency relation due to the difference in the growth factor. Indeed, consider those configurations in which m-galaxies are at $r \gtrsim R_0$ and $(n-m)$ are at $r \lesssim R_0$, the consistency relation will be written as

$$\frac{\langle\delta_{\rm g}(\vec{q},\tau)\delta^{>}_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta^{>}_{\rm g}(\vec{k}_m,\tau_m)\delta^{<}_{\rm g}(\vec{k}_{m+1},\tau_{m+1})\cdots\delta^{<}_{\rm g}(\vec{k}_n,\tau_n)\rangle'_{q\to 0}}{\langle\delta^{>}_{\rm g}(\vec{k}_1,\tau_1)\cdots\delta^{>}_{\rm g}(\vec{k}_m,\tau_m)\delta^{<}_{\rm g}(\vec{k}_{m+1},\tau_{m+1})\cdots\delta^{<}_{\rm g}(\vec{k}_n,\tau_n)\rangle'}$$
$$= -\left(\sum_{a=1}^{m}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\frac{\langle\delta_{{\rm g},L}(\vec{q},\tau)\delta^{>}_{L}(\vec{q},\tau_a)\rangle}{1+2\alpha^2\epsilon}+\sum_{a=m+1}^{n}\frac{\vec{q}\cdot\vec{k}_a}{q^2}\big\langle\delta_{{\rm g},L}(\vec{q},\tau)\delta^{<}_{L}(\vec{q},\tau_a)\big\rangle\right). \tag{5.31}$$

Notice that the right-hand side for the configuration (5.31) is not vanishing even for correlators at equal time for the n-points. This is due to the fact that the long wavelength chameleon-like field correlates only with the overdensity located in the unscreened region, the one in the screened region being completely independent from the chameleon-like perturbation. For instance, for $n=2$ and $m=1$, the corresponding bispectrum reads

$$\boxed{\begin{aligned}&\frac{\langle\delta_{\rm g}(\vec{q},\tau)\delta^{>}_{\rm g}(\vec{k}_1,\tau)\delta^{<}_{\rm g}(\vec{k}_2,\tau)\rangle'_{q\to 0}}{\langle\delta^{>}_{\rm g}(\vec{k}_1,\tau)\delta^{<}_{\rm g}(\vec{k}_2,\tau)\rangle'}\\ &\quad=\left[\frac{2\alpha^2\epsilon}{1+2\alpha^2\epsilon}\big\langle\delta_{{\rm g},L}(\vec{q},\tau)\bar{\delta}_L(\vec{q},\tau)\big\rangle-\big\langle\delta_{{\rm g},L}(\vec{q},\tau)\Delta\delta_L(\vec{q},\tau)\big\rangle\right]\frac{\vec{q}\cdot\vec{k}_1}{q^2}\end{aligned}} \tag{5.32}$$

where $\bar{\delta}_L$, $\Delta\delta_L$ are the formal quantities $\bar{\delta}_L=(\delta^{<}_L+\delta^{>}_L)/2$ and $\Delta\delta_L=(\delta^{<}_L-\delta^{>}_L)$. The latter is also suppressed by $\alpha^2\epsilon$ which therefore gives an estimate of the violation of the "large-scale EP".

Consider now, for instance, a cluster of galaxies of mass $M\sim 10^{14.5\text{–}15}\,M_\odot$ and radius $R_0\sim$ 2–10 Mpc. Inside it $\Phi_{\rm cl}=-GM/R_0\sim -10^{-5}$ and one has [39]

$$\frac{\bar{\varphi}}{2\alpha}\ll 10^{-6}\lesssim\frac{GM}{R_0}, \tag{5.33}$$

where $\bar{\varphi}$ is the asymptotic background value of the scalar φ and the upper bound comes from the solar system [40]. In such a dense object the scalar field is screened, $\epsilon\simeq -\bar{\varphi}/(2\alpha\Phi_{\rm cl})\lesssim 10^{-1}$, and we may take $(n-m)$ galaxies residing there. Away from the cluster there might be small m galaxies with $\Phi_{\rm g}\sim -10^{-8}$ which are unscreened (therefore preferably residing in voids) and $\epsilon\simeq 1$. For this configuration, one expects to see a violation of the consistency relation as predicted by Eq. (5.32).

Notice also that our considerations hold as long as the Compton wavelength m^{-1} associated to the chameleon-like field is larger than the scale where perturbations may be considered in the linear regime. At redshift $z=0$, there is a strong upper bound of about 1 Mpc on such Compton wavelength $m^{-1}(a_0)$ coming from the solar system tests [41,42], implying that the

[3] At the boundary between the over-dense region and the exterior waves of the scalar field will be generated and might propagate both to the interior and exterior. We will ignore these effects since we expect the scalar field to have small oscillations around the static solution deep inside the screened region, and in the unscreened region we expect the scalar field to go to a constant far from the boundary. Close to the boundary our results might not apply, but one can expect even larger violations to the consistency relation due to the gradient of the scalar field being large.

desired effects on the large scale structure are restricted to non-linear scales. However, at higher redshifts a Compton wavelength of the form $m^{-1}(a) = m^{-1}(a_0)(a/a_0)^p$ with $p < -3$ satisfies the experimental constraints and can lead to a modified gravity regime on large linear scales [41]. This scaling of m is faster than the one deduced from the Lifshitz scaling of the scale $k_{\rm NL}(a) \sim a^{-2/(n+4)}$ (during matter domination) at which cosmological perturbations become non-linear [16,43] and the condition $m(a) < k_{\rm NL}(a)$ is easily attained going back in time.

An interesting question is how well one can constrain these theories through the galaxy consistency relation. Though an accurate estimate is beyond the scope of this paper, let us try to make a simple back-of-the-envelope computation by noting that the form of the bispectrum (5.32) is almost the same one obtains in the galaxy local bias model in the presence of a primordial local non-Gaussianity [44] (see also Ref. [17]). Supposing that the combination $\alpha^2\epsilon$ is smaller than unity, one needs basically to identify (barring coefficient of order unity and assuming redshift $z = 0$) $\alpha^2\epsilon(\vec{q} \cdot \vec{k}_1)$ with $f_{\rm NL} H_0^2$, where $f_{\rm NL}$ is the non-linear coefficient parametrizing the level of non-Gaussianity and H_0 is the present Hubble rate. The Fisher matrix analysis applied to the galaxy (reduced) bispectrum performed in Ref. [44] has shown that one can measure $f_{\rm NL}$ up to $\mathcal{O}(10)$ for $k_1 \sim k_{\rm max} \sim 0.1h\ {\rm Mpc}^{-1}$, being $k_{\rm max}$ the smallest scale included in the analysis. Therefore, again very roughly, we expect to be able to measure the modification of gravity of the type we consider at redshifts $z \gtrsim 1$ of the order of $\mathcal{O}(10)(H_0/k_{\rm max}) \sim 10^{-3}(0.1h\ {\rm Mpc}^{-1}/k_{\rm max})$, where we have taken $q \sim 10^2 H_0$.

Similar considerations apply also to more conventional modifications of gravity induced by scalars, like Brans–Dicke theory, or dilaton gravity. In these theories, in spite of the fact that there is a universal coupling of the scalar to matter, there is a violation of the *strong* EP because a gravitational experiment can yield different results in different points in spacetime. However, this violation is subleading in the post-Newtonian approximation for non-relativistic matter and it can only give order one effects in strongly bound systems as binary systems and black holes [45]. To be more precise here, let us consider an action of the general form

$$S = \int {\rm d}^4x \sqrt{-g}\big(f(R,\phi,X) + \mathcal{L}_{\rm m}\big), \tag{5.34}$$

where $f(R,\phi,X)$ is a function of the Ricci scale R, a scalar ϕ and its kinetic term and $X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi$. This form of the action describes many models of modify gravity like Brans–Dicke theory, dilaton gravity, $f(R)$ and many others. In this general class of models, the non-relativistic matter still satisfies Eqs. (2.4)–(2.6), where now $\Omega_{\rm m} = 8\pi G_{\rm eff}\bar{\rho}a^2/3\mathcal{H}^2$ and $G_{\rm eff}$ is an effective Newton constant which encodes the modification of gravity given by [46]

$$G_{\rm eff}(\tau) = \frac{1}{8\pi F}\frac{f_{,X} + 4\big(f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F}\big)}{f_{,X} + 3\big(f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F} + \frac{F_{,\phi}^2}{F}\big)}, \qquad F = \frac{\partial f}{\partial R}. \tag{5.35}$$

Therefore, when

$$f_{,X}\frac{k^2}{a^2}\frac{F_{,R}}{F} \ll 1, \tag{5.36}$$

the effective Newton constant is only time dependent and it just modifies the temporal dependence of the local growth function of the overdensity evolution. In this case, still, one may generate a long wavelength velocity mode by a vector $\vec{n}(\tau)$ as in Eq. (2.15). In the opposite case, Eq. (5.36) is not satisfied and $G_{\rm eff}$ turns out to be space-dependent. The overdensity δ turns out to be also space-dependent as well and there may be no $\vec{n}(\tau)$ to generate a long wavelength velocity mode within the sphere of radius R_0. To see when this is possible, let us mention that there is a crossover scale when the k-dependence of $G_{\rm eff}$ starts become strong and which is defined by

$$R_0 = \frac{a}{k} \approx \left(\frac{F_{,R}}{F} f_{,X} \right)^{1/2}. \tag{5.37}$$

If $R \lesssim R_0$, one may still define $\vec{n}(\tau)$ and so long wavelength modes may be generated. On the other side if $R \gtrsim R_0$, i.e. modification of gravity appears within the sphere, then there is no globally defined $\vec{n}(\tau)$ inside the sphere of radius R, which will cause a modification of the consistency relation. So the lesson here is that violation of the consistency relations is a signal of the spatial dependence of the effective Newton constant $G_{\rm eff}$ and of a modification of gravity at large scales.

We should also note that we have not considered here intrinsic violation of the EP, i.e. at the microscopic level [47–50]. One for example may consider the case of extra scalar, vector or tensor couplings to only one component, say baryonic matter or dark matter. It has been recently realized that in the modified gravity models where there is an efficient screening phenomenon to make the set-up experimentally consistent there might also be order unity violation of the EP [39]. Such possibilities has been considered recently in Ref. [51] where it has been pointed out the interesting feature that if a large scale velocity bias exists between the different components new terms appear in the consistency relations with respect to the single species case.

6. Conclusions

In this paper we have discussed the implications of the symmetry enjoyed by the Newtonian equations of motion describing the dark matter and galaxy fluids coupled through gravity. The fact that such symmetry applies to both galaxies and dark matter is particularly welcome because one can reach conclusions which are independent from the galaxy bias. On the contrary, one can use the power of the symmetry to deduce relevant informations on the theory of galaxy bias. In particular, we have shown that an unavoidable consequence of the symmetries at our disposal is that the bias is expected to be non-local. Furthermore, we have studied the modification (or violation) of the consistence relation in the case in which gravity is modified because of the presence of extra degrees of freedom propagating unscreened at large cosmological distances. Let us reiterate that our results are based on the assumption that the galaxy number is conserved. Eventually, one would like to extend our considerations by accounting for phenomena like halo formation and merging, nevertheless if the modification in the proper equations are such that the symmetries studied in this paper are preserved, e.g. if the new terms are a local function of the dark matter density, then our considerations remain valid. Also, apart from applying to non-linear scales and directly to galaxies, our results have the virtue of not being sensitive to the single stream approximation and to be valid also in the presence of velocity bias and/or vorticity (which is generated at higher-order in perturbation theory). Therefore, assuming that primordial perturbations satisfy the consistency relations of [8], the observation of a deviation from the consistency relation for the bispectrum of galaxies, Eq. (2.21), would signal either the inapplicability of the Eulerian bias model even including “non-local” terms as in Eq. (3.6) or the violation of the EP in the underlying theory of gravity.

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When completing this work, Ref. [51] appeared. Our results, when overlap is possible, agree with theirs. We thank M. Pietroni and M. Peloso for useful correspondence. We acknowledge related work by P. Creminelli, J. Gleyzes, M. Simonović and F. Vernizzi and thank them for spotting an omission in the consistency relation in redshift space in an earlier version of this draft. H.P., J.N. and A.R. are supported by the Swiss National Science Foundation (SNSF), project "The non-Gaussian Universe" (project number: 200021140236). The research of A.K. was implemented under the "Aristeia" Action of the "Operational Programme Education and Lifelong Learning" and is co-funded by the European Social Fund (ESF) and National Resources. It is partially supported by European Union's Seventh Framework Programme (FP7/2007–2013) under REA grant agreement No. 329083.

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